3/11/2015

محاضرة [4] تصميم ماكينات

AMR M.٢

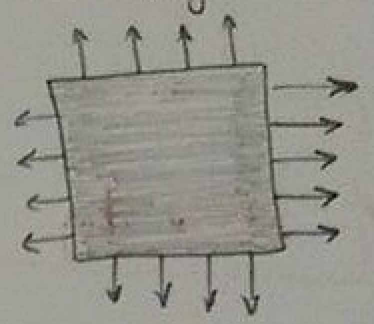
[9]

plan strain

$$\boxed{E = \frac{\sigma_x}{\epsilon_x}}$$

$$\boxed{n = \frac{\text{lateival strain}}{\text{long. strain}}}$$

$$\boxed{\epsilon_x = \frac{\sigma_x}{E}} \qquad \boxed{\epsilon_y = \epsilon_z = -n \cdot \epsilon_x}$$



[a] Given σ_x & σ_y

Find ϵ_x & ϵ_y

$$\epsilon_x = \frac{\sigma_x}{E} - n\frac{\sigma_y}{E} \rightarrow [1]$$

$$\epsilon_y = \frac{\sigma_y}{E} - n\frac{\sigma_x}{E} \rightarrow [2]$$

[b] Given ϵ_x & ϵ_y

Find σ_x & σ_y

$$\sigma_x = \frac{E}{(1-n^2)}(\epsilon_x + n\,\epsilon_y) \rightarrow [3]$$

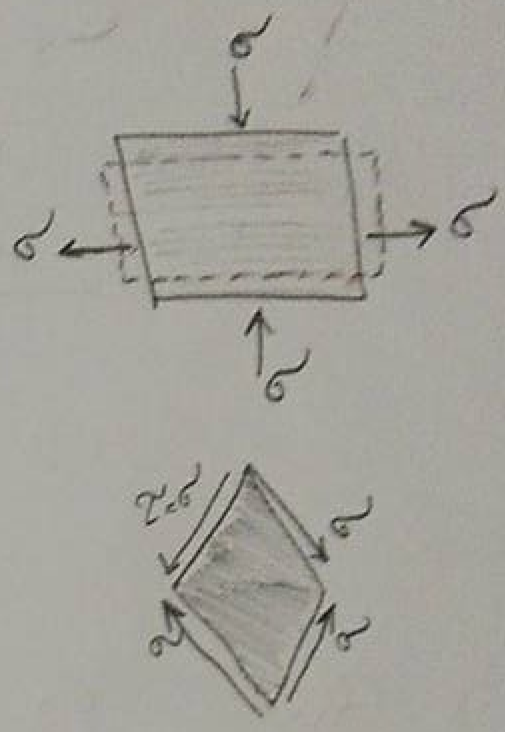
$$\sigma_y = \frac{E}{(1-n^2)}(\epsilon_y + n\,\epsilon_x) \rightarrow [4]$$

[b]

$$\epsilon_s = \gamma$$

shear strain

$$\tau = G.\gamma$$



$$\boxed{\epsilon_x = \frac{\sigma}{E} + n\frac{\sigma}{E} = \frac{\sigma}{E}(1+n)}$$

$$\boxed{\epsilon_y = \frac{-\sigma}{E} - n\frac{\sigma}{E} = \frac{-\sigma}{E}(1+n)}$$

For case (a)

$$w.D = 2 * \tfrac{1}{2}\sigma.\epsilon = \sigma.\epsilon$$

For case (b)

$$\gamma = \frac{\tau}{G} = \frac{\sigma}{G}$$

$$w.D = \tfrac{1}{2}\tau.\gamma = \tau^2/2G$$

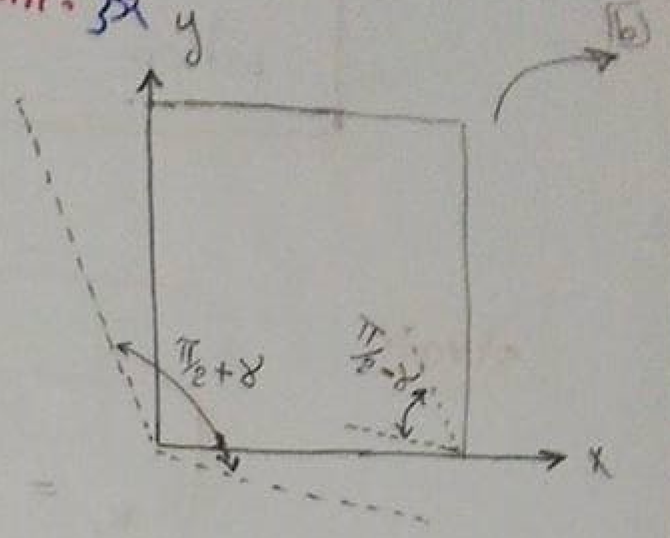
$$\sigma.E = \frac{\tau^2}{2G}$$

$$\frac{\tau^2}{2G} = \frac{\sigma^2}{E}(1+n) = \frac{\sigma^2}{2G}$$

$$\boxed{\therefore G = \frac{E}{2(1+n)}}$$

c

Analogy between stresses & strain.



stresse		strain
ϵ_x	$\longrightarrow$	σ_x
ϵ_y	$\longrightarrow$	σ_y
$\frac{1}{2}\gamma_{xy}$	$\longrightarrow$	τ
α	$\longrightarrow$	θ

$$\boxed{\sigma_\theta = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta.}$$

For max. stress.

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}}$$

$$\boxed{\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\boxed{\tau_\theta = \frac{-(\sigma_x-\sigma_y)}{\;}\sin 2\theta + \tau_{xy}\cdot\cos 2\theta}$$

For max. τ_θ

$$\boxed{\tan 2\theta_s = \frac{-(\sigma_x-\sigma_y)}{2\tau}}$$

$$\boxed{\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

d

$$\boxed{\text{where } \sigma_{1s} = \sigma_{2s} = \left(\frac{\sigma_x + \sigma_y}{2}\right)}$$

1 shear 2 shear

strain

$$\epsilon_\alpha = \frac{(\epsilon_x + \epsilon_y)}{2} + \frac{(\epsilon_x - \epsilon_y)}{2} \cos 2\alpha + \cdots$$

$$\cdots + \frac{1}{2} \gamma_{xy} \cdot \sin 2\alpha$$

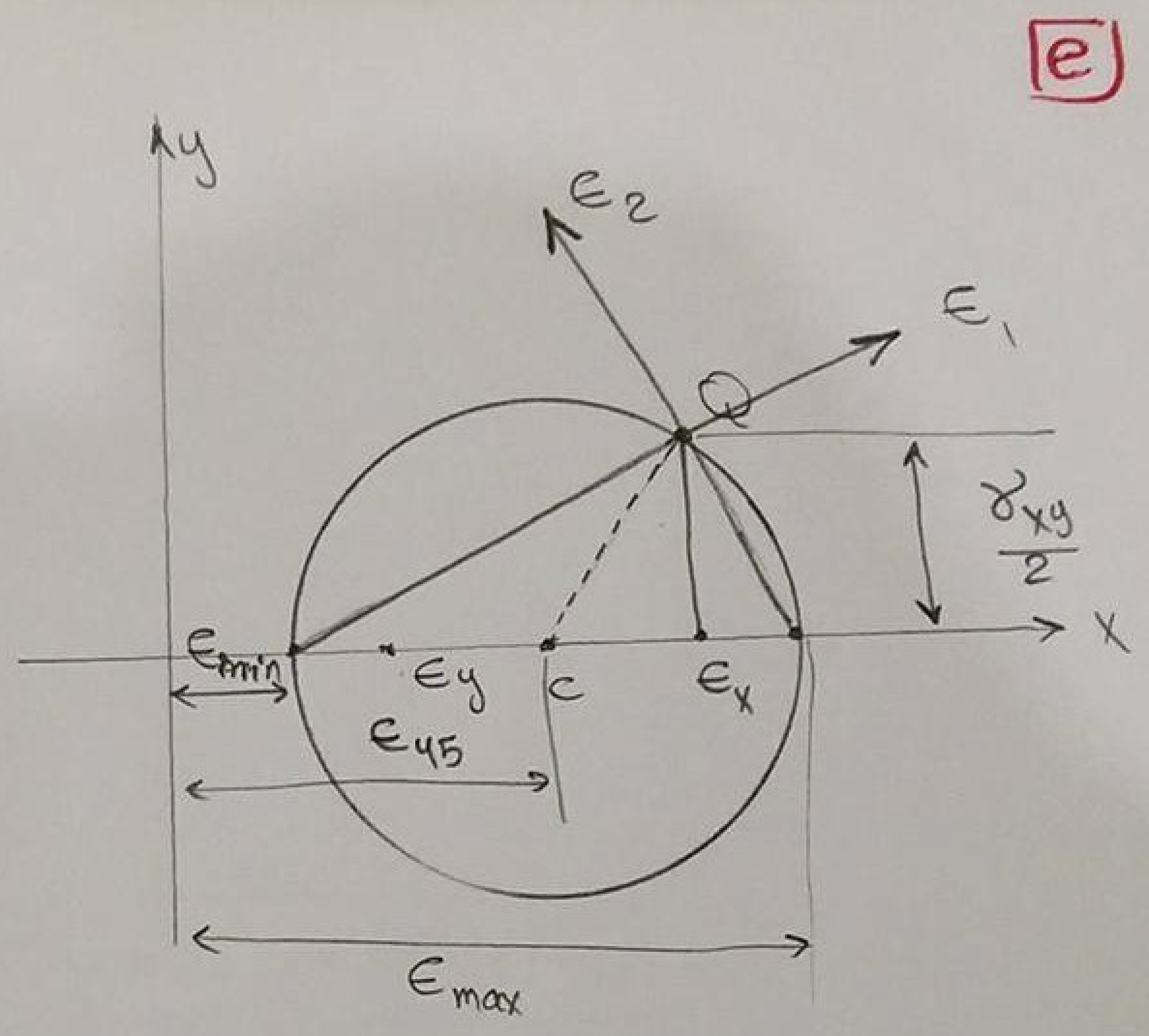
$$\gamma_\alpha = -(\epsilon_x + \epsilon_y) \sin 2\alpha + \frac{1}{2} \gamma_{xy} \cos 2\alpha$$

$$\epsilon_{1/2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{(\epsilon_x + \epsilon_y)^2 + \gamma_{xy}^2}$$

$$\gamma_{max} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\epsilon_1 = \frac{E}{(1-n^2)} (\epsilon_1 + n\, \epsilon_2)$$

$$\epsilon_2 = \frac{E}{(1-n^2)} (\epsilon_2 + n \cdot \epsilon_1)$$



$$\gamma_{max} = \epsilon_1 - \epsilon_2$$